Exercise 24

If, in Example 4, one molecule of the product C is formed from one molecule of the reactant A and one molecule of the reactant B, and the initial concentrations of A and B have a common value $[\mathrm{A}] = [\mathrm{B}] = a$ moles/L, then

$$[\mathrm{C}] = a^2kt/(akt+1)$$

where k is a constant.

(a) Find the rate of reaction at time t.

(b) Show that if $x = [\mathrm{C}]$, then

$$\frac{dx}{dt} = k(a-x)^2$$

(c) What happens to the concentration as $t \to \infty$?

(d) What happens to the rate of reaction as $t \to \infty$?

(e) What do the results of parts (c) and (d) mean in practical terms?

Solution

Part (a)

The rate of reaction is the derivative of the concentration at time t.

$$\begin{aligned}\frac{d[\mathrm{C}]}{dt} &= \frac{d}{dt}\left(\frac{a^2kt}{akt+1}\right)\\
&= \frac{\left[\frac{d}{dt}(a^2kt)\right](akt+1) - \left[\frac{d}{dt}(akt+1)\right](a^2kt)}{(akt+1)^2}\\
&= \frac{(a^2k)(akt+1)-(ak)(a^2kt)}{(akt+1)^2}\\
&= \frac{a^2k}{(akt+1)^2}\end{aligned}$$

Part (b)

Let $x = [\mathrm{C}]$.

$$\begin{aligned}\frac{dx}{dt} = \frac{a^2k}{(akt+1)^2} = k\left(\frac{a}{akt+1}\right)^2 &= k\left(\frac{a+a^2kt-a^2kt}{akt+1}\right)^2\\
&= k\left[\frac{a(1+akt)-a^2kt}{akt+1}\right]^2\\
&= k\left(a-\frac{a^2kt}{akt+1}\right)^2\end{aligned}$$

Therefore,

$$\frac{dx}{dt} = k(a - x)^2.$$

Part (c)

Take the limit of the concentration as $t \to \infty$.

$$\lim_{t\to\infty} [\mathrm{C}] = \lim_{t\to\infty} \frac{a^2kt}{akt + 1} = \lim_{t\to\infty} \frac{a^2k}{ak + \frac{1}{t}} = \frac{a^2k}{ak + 0} = a$$

Part (d)

Take the limit of the rate of reaction as $t \to \infty$.

$$\lim_{t\to\infty} \frac{d[\mathrm{C}]}{dt} = \lim_{t\to\infty} \frac{a^2k}{(akt + 1)^2} = \frac{a^2k}{(\infty)^2} = 0$$

Part (e)

The fact that $\lim_{t\to\infty} [\mathrm{C}] = a$ means that the final product concentration will be the same as the initial reactant concentration. The fact that $\lim_{t\to\infty} \frac{d[\mathrm{C}]}{dt} = 0$ means that the chemical reaction will finish only after an infinite amount of time has passed.